Advanced Robust Control Theory and Its Applications

Robust Control: Theory and Applications (textbook)

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Contents of lecture

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- Basics of convex analysis and LMI
- Review of linear systems
- Parameterization of stabilizing controllers
- KYP lemma
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- Robustness analysis 2: Lyapunov method
- Robustness analysis 3: IQC approach

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- μ synthesis
- Regional pole placement
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- Textbook: Robust Control: Theory and Applications, K.Z Liu and Y. Yao, Wiley International (2016.10)

Chapter 1

Engineering background and major methodologies

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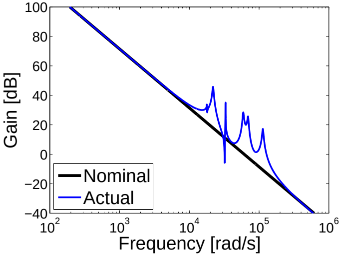
3. A Brief History of Robust Control

Engineering Background: HDD

- Hard disk is commonly used as the data storage for computers.
- The rigid body model is a double-integrator (solid line).
- The arm has numerous resonant modes in the high frequency band which vary with the manufacturing error.
- Need to design the control system based on the rigid body model.

(a) Photo

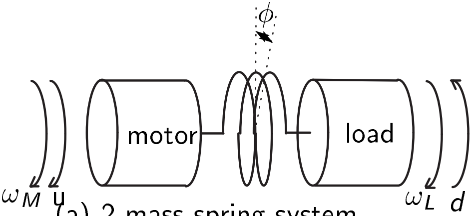


(b) Bode plot

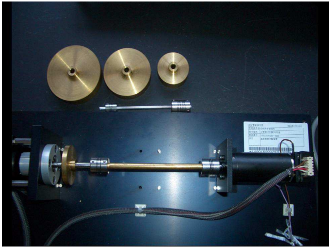
Figure: Hard disk drive and its Frequency response

Engineering Background: 2-mass-spring system

- Essentially a motor drive system in which the motor and load are connected through a shaft.
- Purpose: control of the load speed indirectly by controlling the motor inertia moment.
- Typical examples: DVD drive in home electronics, rolling mill in steel factory.
- Load inertia and shaft spring constant vary with application.



(a) 2-mass-spring system



(b) Motor drive

Engineering Background: 2-mass-spring system

- J_M: motor inertia moment, k: shaft spring constant, J_L:load inertia moment,
- ω_M: motor speed, ω_L: load speed, ϕ: torsional angle of shaft, u: motor torque, d: torque disturbance acting on the load.
- Equations of moment balance and speed

$$\begin{aligned} &J_L\dot{\omega}_L + D_L\omega_L = k\phi + d \\ &\dot{\phi} = \omega_M - \omega_L \\ &J_M\dot{\omega}_M + D_M\omega_M + k\phi = u. \end{aligned}$$

- State equation: $x = [\omega_L \ \ \phi \ \ \omega_M]^T$

$$\dot{x} = \begin{bmatrix} -\frac{D_L}{J_L} & \frac{k}{J_L} & 0 \\ -1 & 0 & 1 \\ 0 & -\frac{k}{J_M} & -\frac{D_M}{J_M} \end{bmatrix} x + \begin{bmatrix} \frac{1}{J_L} \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J_M} \end{bmatrix} u \qquad (1)$$

$$y = \omega_M = [0 \ \ 0 \ \ 1]x. \qquad (2)$$

Adverse impact of model uncertainty

- Simplified dynamics of HDD

$$\tilde{P}(s) = \frac{1}{s^2} - \frac{0.1 \times \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad \zeta = 0.02,\ \omega_n = 5.$$

- Design based on rigid body model $P(s) = 1/s^2$: damping ratio $1/2$, natural frequency 4 [rad/sec]
- Characteristic polynomial $p(s) = s^2 + 4s + 16$
- PD compensator $K(s) = 4(s+4)$
- Characteristic polynomial of the actual closed-loop

$$\tilde{p}(s) = s^4 - 5.8s^3 + 1.8s^2 + 103.2s + 400$$

  Closed-loop poles

$$5.2768 \pm j3.8875, \quad -2.3768 \pm j1.9357,$$

- Closed-loop system not only fails to achieve performance improvement, but also losses the stability (spillover).

Adverse impact of model uncertainty

- Reason: Roll-off of controller gain is not sufficient near $\omega_n = 5$ which excites the resonant mode.

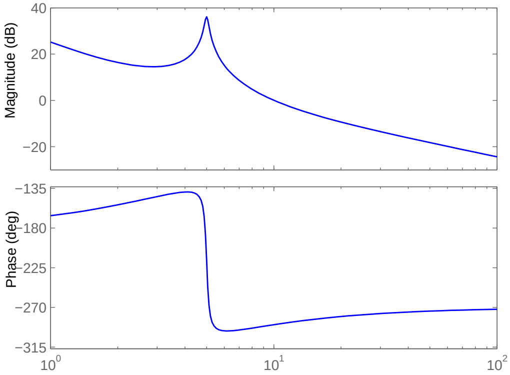


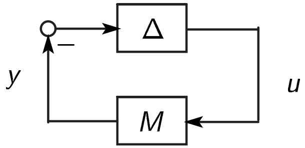
Figure: Bode plot of open loop transfer function

Methodologies of Robust Control

1. Fundamental of robust control
2. Small gain approach
3. Positive real method
4. Lyapunov method
5. Robust regional pole placement
6. Gain-scheduling

Fundamentals of robust control

- Nyquist stability criterion
  Suppose that $M(s), \Delta(s)$ are stable. Then, the closed loop system is stable iff the loop gain $M(j\omega)\Delta(j\omega)$ does not encircle the critical point $(-1, j0)$.
  1. Small-gain: $|M(j\omega)\Delta(j\omega)| < 1$ for all frequencies
  2. Passivity: $\arg M(j\omega) + \arg \Delta(j\omega) \neq \pm 180^\circ$ for all frequencies



Small Gain Approach

- System model: $P(s)$, Actual system: $\tilde{P}(s)$
- Simplest uncertainty model

$$\Delta(s) := \tilde{P}(s) - P(s) \Leftrightarrow \tilde{P}(s) = P(s) + \Delta(s). \tag{3}$$

- Uncertainty modeling: gain bound

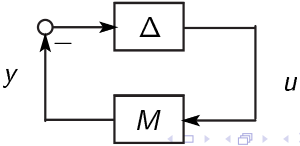
$$0 \leq |\Delta(j\omega)| \leq |W(j\omega)| \ \forall\omega. \tag{4}$$

  $W(s)$ is a bounding function of uncertainty $\Delta(s)$
- Stability condition (*small gain condition*)

$$|M(j\omega)\Delta(j\omega)| < 1 \ \forall\omega \ \Leftrightarrow \ |M(j\omega)W(j\omega)| < 1 \ \forall\omega \tag{5}$$

- Loop gain $L(s) = M(s)\Delta(s)$ never encircles $(-1, j0)$

$$M(s) = \frac{K}{1 + PK}$$



Positive Real Method

- Uncertainty modeling: phase bound

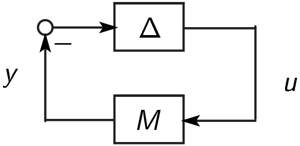
$$-90^\circ \le \arg \Delta(j\omega) \le 90^\circ \ \forall \omega \ \Leftrightarrow \ \Re[\Delta(j\omega)] \ge 0 \ \forall \omega \tag{6}$$

- Stability condition (*positive real condition*)

$$-90^\circ < \arg M(j\omega) < 90^\circ \ \forall \omega \ \Leftrightarrow \ \Re[M(j\omega)] > 0 \ \forall \omega. \tag{7}$$

- Phase angle $L(s) = M(s)\Delta(s)$ is never equal to $\pm 180^\circ$ s.t. $L(j\omega)$ does not encircle the critical point $(-1, j0)$
- *Positive real function*: a transfer function satisfying $\Re[G(j\omega)] \ge 0 \ \forall \ \omega$

$$M(s) = \frac{K}{1 + PK}$$



Lyapunov Method

- Basic idea of Lyapunov stability theory

$$\dot{x} = Ax, \quad x(0) \neq 0.$$

Quadratic Lyapunov function (Energy)

$$V(x) = x^T Px, \quad P > 0$$

When the energy is strictly decreasing

$$\dot{V}(x) < 0 \quad \forall x(0) \neq 0 \Rightarrow \lim_{t \to 0} V(x(t)) = 0 \Rightarrow \lim_{t \to 0} x(t) = 0$$

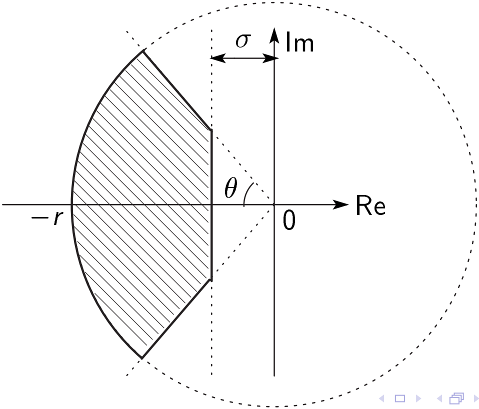
Stability condition

$$A^T P + PA < 0.$$

- Effective in handling parameter uncertainty due to its state space character
- Even if A is uncertain, the stability is secured so long if there exists a constant matrix $P > 0$ satisfying this inequality (*Quadratic Stability*).

Robust Regional Pole Placement

1. Response speed is determined by poles' positions
2. For a system with parameter uncertainty, it is not possible to place its poles at specified points
3. However, placing its poles in a region on the complex plane is possible



Gain-Scheduling

- Response design is difficult for nonlinear systems
- Many nonlinear systems can be described as linear systems with time-varying parameters, called *linear parameter varying* (LPV).
- When the states contained in the parameters of LPV system can be measured online, the time-varying parameters can be computed.
- Let the controller parameters vary with the time-varying parameters, it is possible to obtain a better control performance.

1-link arm

Motion equation

$$J\ddot{\theta} + mgl\sin\theta = u.$$

Set $p(t) = \sin\theta/\theta$, then

$$J\ddot{\theta}(t) + mglp(t)\theta(t) = u(t).$$

LPV system with a bounded time-varying parameter $p(t)$

$$|p(t)| = \left|\frac{\sin\theta(t)}{\theta(t)}\right| \le 1$$

When we use the control input

$$u(t) = mglp(t)\theta(t) - 2\zeta\omega_n J\dot{\theta}(t) - \omega_n^2 J\theta(t),$$

Characteristic polynomial of CLS becomes

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0.$$

By adjusting ζ, ω_n, we can easily achieve a high performance.
In fact, the first term is a nonlinear term $mgl\sin\theta$.

A Brief History of Robust Control

1. Indirect consideration (Up to 1960's)
   Stability margins (gain margin, phase margin) against model uncertainty in the high frequency domain, online tuning
2. Modern control theory: uncertainty excluded (1960's-1970's)
   Pole placement, LQG (least quadratic Gaussian) optimal control, direct implementation was rare, low-pass filter always needed.
3. Sensitivity analysis (Cruze, 1960's)
   Relative change of closed loop transfer function

$$H(s) = \frac{L(s)}{1 + L(s)}, \quad \lim_{\Delta L \to 0} \frac{\Delta H / H}{\Delta L / L} = \frac{1}{1 + L} := S(s)$$

   Valid only for very small perturbations, has nothing to do with uncertainty dynamics.

A Brief History of Robust Control

1. 1970's-1980's
   Zames ($\mathcal{H}_\infty$ norm measurement of system performance, sensitivity analysis) and Doyle-Stein (small gain principle). Both believe that the model uncertainty should be modeled by its gain bound.
2. $\mathcal{H}_\infty$ control (1980's)
   Operator theory (Zames, Francis, Doyle), Nevanlinna-Pick interpolation theory (Kimura), state space approach (Doyle), Riccati Equation solution (Doyle-Glover, DGKF)
3. LMI (linear matrix inequality) approach (1990's)
   French school (Gahinet, et al.): $\mathcal{H}_\infty$ control, regional pole placement, gain-scheduling
4. Others
   Introduction of uncertainty phase (Haddad-Berstein, Tits, Peterson),
5. 21st century
   Liu (ultimate robust performance design, utilization of uncertainty gain and phase, input/output approach)